Chapter 11

Antenna Diversity

We have seen that in a multipath propagation environment, fast fading due to cancellation of multipath signals leads to low power at the receiver and causes interruption of the communication channel. There are a number of approaches to overcoming fading, including coding schemes, interleaving to spread single bit errors over a long time window, and increasing the transmitter power. These approaches are more or less indirect. Diversity is a class of techniques which address fading and increase channel communications capacity in a more direct way.

The basic principle of diversity is to receive the same signal at multiple times, locations, or frequencies so that if one received signal is in a fade, another is available with larger signal power. In communication theory, it is common to refer to the different signals as channel branches. There are several types of signal diversity:

Spatial: Multiple antennas at different locations.

Angle: Multiple antennas with different gain patterns.

Polarization: Two antennas receiving orthogonal polarizations.

Time: Transmit the same signal multiple times at an interval larger than the coherence time of the channel.

Frequency: Transmit the same signal at multiple frequencies with spacing larger than the coherence frequency of the channel.

This chapter will focus on antenna diversity. To analyze diversity approaches, from an electromagnetic point of view we need to determine the degree of correlation of the received signals at multiple antennas. The lower the correlation, the more likely one branch has a high SNR when another has low SNR, so for diversity applications, low correlation is desirable. From a signal processing point of view we need algorithms that exploit multiple transmissions of the same signal or bit sequence to increase the channel availability.

11.1 Signal Correlation in a Multipath Environment

In order to analyze the performance of a given approach to antenna diversity, we need to understand the degree of correlation between signals received by multiple antennas. The array output voltage correlation matrix defined in Section 8.3 provides a mathematical framework for analyzing signal correlation.

For an array with complex baseband output voltages arranged into the column vector $\mathbf{v}(t)$, the correlation matrix is

$$\mathbf{R} = \mathrm{E}[\mathbf{v}(t)\mathbf{v}(t)^H] \tag{11.1}$$

Since we are typically interested in the relative correlation and not on the power contained in the signals from the array elements, it is convenient to define a normalized correlation coefficient according to

$$\rho_{mn} = \frac{R_{mn}}{\sqrt{R_{mm}R_{nn}}} = \frac{\mathrm{E}[v_m(t)v_n^*(t)]}{\mathrm{E}[|v_m(t)|^2]^{1/2}\mathrm{E}[|v_n(t)|^2]^{1/2}} \tag{11.2}$$

The correlation coefficient satisfies $0 \le |\rho_{mn}| \le 1$. If $v_m(t) = v_n(t)$ then the signals are fully correlated and $\rho_{mn} = 1$. If the signals are proportional, they are also fully correlated. If $v_m(t) = \alpha v_n(t)$, where α is an arbitrary complex constant, then $\rho_{mn} = \alpha/|\alpha|$ and $|\rho_{mn}| = 1$. As a matrix, $\boldsymbol{\rho}$ is similar to the array correlation matrix $\mathbf{R}$ but has diagonal elements equal to unity. For a given pair of elements, it is common to drop the subscripts and refer to the correlation coefficient simply as ρ.

If two array output signals are highly correlated ($\rho \to 1$), then the signals do not contain much more information than the signal from one antenna, and regardless of the signal processing scheme used with the multiantenna system, the achievable diversity gain is small. If the correlation is low ($\rho \to 0$), then if one branch is in a fade, it is likely that the other signal is strong and has a high enough SNR for reliable detection. The goal now is to analyze the correlation coefficient for various types of diversity.

11.2 Spatial Diversity

Spatial diversity is the use of an array of identical elements with different locations to overcome fades and improve channel capacity. For spatial diversity to be useful, we need the signals from multiple antennas to be uncorrelated, so that if the local power drops at one antenna due to fading the power remains large at another. Since the correlation coefficient depends on the angles of arrival of the multipaths as well as the array configuration, we will consider several specific angle of arrival distributions.

11.2.1 Uniform Spherical Arrival Angle Distribution

From (8.25), the array open circuit voltage correlation matrix for a single incident plane wave with power density S^{sig} is

$$\mathbf{R}_{\mathrm{sig,oc}} = c_2 S^{\mathrm{sig}}\mathbf{E}_p(\overline{r})\mathbf{E}_p^H(\overline{r}) \tag{11.3}$$

In a multipath environment, the signal correlation matrix can be obtained by integrating (11.3) over the probability distribution function of the incoming multipaths. This leads to

$$\mathbf{R}_{\mathrm{sig,oc}} = c_2 \int S^{\mathrm{sig}}\mathbf{E}_p(\overline{r})\mathbf{E}_p^H(\overline{r}) p(E^{\mathrm{sig}}, \hat{p}, \Omega)\, dE^{\mathrm{sig}}\, d\hat{p}\, d\Omega \tag{11.4}$$

where $p(E^{\mathrm{inc}}, \hat{p}, \Omega)$ is the joint PDF of the incoming waves as a function of amplitude, polarization, and angle of arrival.

If the PDF is uniform with respect to polarization and angle of arrival, then the integral in (11.4) becomes identical to the integral in the external thermal noise correlation matrix (8.26) for a uniform brightness temperature distribution. It follows that $\mathbf{R}_{\mathrm{sig,oc}}$ is proportional to the correlation matrix of an isotropic external thermal noise distribution, which is in turn proportional to the array element pattern overlap matrix $\mathbf{A}$. If the array is lossless, we arrive at the remarkable result that the signal correlation matrix in a propagation environment for which all polarizations and angles of arrival are equally likely, the isotropic thermal noise correlation matrix, the element pattern overlap matrix, and the array mutual resistance matrix are all identical up to a scale factor!

From (7.24), the correlation coefficient for two isotropic antennas separated by a distance d is

$$\rho_{12} = \frac{\sin(kd)}{kd} \tag{11.5}$$

This result implies that for uniformly distributed arrival angles, to obtain low correlation between the received signals at two antennas, the spacing should be an integer multiple of $\lambda/2$, or the spacing should be large enough that ρ_{12} is small due to the denominator of (11.5).

11.2.2 Horizontal Arrival Angle Distribution

In terrestrial communications systems, antenna height above ground is relatively small, and the distribution of incoming waves at a receiver can be approximated as consisting only of horizontal arrival angles. The correlation coefficient for two isotropic or omnidirectional antennas separated by a distance d in a uniformly distributed horizontal multipath environment is

$$\begin{aligned}\rho_{12} &= \frac{1}{2\pi}\int_0^{2\pi} e^{jkd\cos\phi}\,d\phi \\ &= J_0(kd) \qquad (11.6)\end{aligned}$$

In this case, the smallest spacing which leads to zero correlation is at $kd \simeq 2.4$ or $d \simeq 0.38\,\lambda$. This spacing is often considered to be optimal for spatial antenna diversity.

11.3 Angle Diversity

If two antennas are oriented so that the main beams are in different directions, in a multipath environment they will tend to receive uncorrelated signals. A step function pattern is unrealizable, but can be used to estimate the degree of correlation in terms of the pattern overlap. If we consider two antennas with step function azimuthal radiation patterns of width θ_0 that overlap by an angle θ_r, the signal correlation coefficient with a horizontal arrival angle distribution is

$$\begin{aligned}\rho_{12} &= \frac{1}{\theta_0}\int_0^{\theta_r} d\phi \\ &= \frac{\theta_r}{\theta_0}\end{aligned}$$

If there is no overlap, $\rho_{12} = 0$, which is optimal for angle diversity. If there is complete overlap, $\rho_{12} = 1$, and there is no benefit in having a second antenna other than possibly an increase in gain. Because high directivity is difficult to obtain with a small antenna, angle diversity is typically not used for a compact system.

11.4 Polarization Diversity

Signals arriving at a receiver in orthogonal polarizations often have low correlation, due to the polarization dependence of scattering in the propagation environment. If we assume that the two polarizations are completely uncorrelated, we can determine the correlation coefficient as a function of the polarization discrimination of two antennas.

Suppose that two elements have radiation patterns with polarizations given by

$$\overline{E}_1 = \beta\hat{\theta} + \hat{\phi}$$
$$\overline{E}_2 = \hat{\theta}$$

The signal correlation matrix is

$$\begin{aligned}\mathbf{R}_{\mathrm{sig,oc}} &\sim \mathrm{E}[\mathbf{E}_p \mathbf{E}_p^H] \\ &= \mathrm{E}\left[\begin{bmatrix}\hat{p}\cdot\overline{E}_1 & \hat{p}\cdot\overline{E}_2\end{bmatrix}\begin{bmatrix}\hat{p}\cdot\overline{E}_1^* \\ \hat{p}\cdot\overline{E}_2^*\end{bmatrix}\right] \\ &= \mathrm{E}\begin{bmatrix}|\beta p_\theta + p_\phi|^2 & (\beta p_\theta + p_\phi)p_\theta \\ p_\theta(\beta p_\theta + p_\phi) & |p_\theta|^2\end{bmatrix}\end{aligned}$$

If we assume that the two polarizations p_θ and p_ϕ are uncorrelated, then

$$\mathbf{R}_{\mathrm{sig,oc}} \sim \begin{bmatrix}\beta+1 & \beta \\ \beta & 1\end{bmatrix} \tag{11.7}$$

from which it can be seen that the correlation coefficient is

$$\rho_{12} = \frac{\beta}{\sqrt{\beta+1}} \tag{11.8}$$

If the polarization discrimination is perfect, $\beta = 0$ and $\rho_{12} = 0$. If β is large, then $\rho_{12} \to 1$, and polarization diversity is not possible.

11.5 Processing Diversity Signals

If some type of diversity is used to obtain M partially correlated outputs or branches, we need a method to combine the signals to make use of diversity to increase the communication channel performance. We will see that there is a fundamental tradeoff between system performance and complexity.

11.5.1 Selection Diversity

A conceptually simple approach to diversity is to design the receiver to select the branch with the highest SNR. In practice, we may not know the SNR in each branch, so we can choose the branch with the largest total power (signal plus noise). We will analyze the performance improvement of this approach for a Rayleigh channel. For the one branch, the probability that the local SNR is less than a given value is given by the CDF

$$F(\gamma_0) = 1 - e^{-\gamma_0/\Gamma} \tag{11.9}$$

The probability that all M branches have SNR less than γ_0 is

$$F_M(\gamma_0) = P(\gamma_1 \le \gamma_0, \gamma_2 \le \gamma_0, \ldots, \gamma_M \le \gamma_0) = (1 - e^{-\gamma_0/\Gamma})^M \tag{11.10}$$

where we assume that the branches are completely uncorrelated ($\rho_{mn} = 0,\ m \ne n$).

Suppose that the SNR must be greater than $\Gamma/10$ to receive a given signal. For one branch,

$$F_1(\Gamma/10) = 1 - e^{-1/10} \simeq 0.1 \tag{11.11}$$

so that the local SNR will be too small 10% of the time. With $M = 2$,

$$F_4(\Gamma/10) = (1 - e^{-1/10})^2 \simeq 0.0091 \tag{11.12}$$

so that the SNR is too small only 1% of the time. This is a significant improvement over a single branch. If the branch signals are correlated, the diversity gain will not be as great, since if one branch fades, it is more likely that other branches also fade.

We can determine the mean SNR for selection diversity in a Rayleigh channel. The PDF is

$$\begin{aligned} f_M(\gamma) &= \frac{d}{d\gamma} F_M(\gamma) \\ &= \frac{M}{\Gamma}(1 - e^{-\gamma/\Gamma})^{M-1} e^{-\gamma/\Gamma} \end{aligned} \tag{11.13}$$

The average SNR is

$$\begin{aligned} \Gamma_M &= \int_0^\infty \gamma f_M(\gamma)\, d\gamma \\ &= \int_0^\infty \frac{M}{\Gamma}(1 - e^{-\gamma/\Gamma})^{M-1} e^{-\gamma/\Gamma}\, d\gamma \\ &= \Gamma \sum_{m=1}^{M} \frac{1}{m} \end{aligned} \tag{11.14}$$

This expression shows that the incremental improvement for additional branches decreases as more branches are added.

11.5.2 Maximum Ratio Combining

A more sophisticated approach to diversity is to use information in more than one branch, rather than only the branch with the largest SNR. To accomplish this, all M signals must be cophased or time shifted to account for differences in the path delays. Once this has been done, the signals can be summed using weights w_n^*. The signal envelope at the output is

$$r_{\mathrm{out}} = \sum_1^M w_m^* r_m \tag{11.15}$$

The noise power is

$$P_{\mathrm{n,out}} = P_{\mathrm{n}} \sum_m |w_m|^2 \tag{11.16}$$

where we have assumed equal noise power P_{n} in each branch. The SNR is

$$\begin{aligned} \gamma_{\mathrm{out}} &= \frac{P_{\mathrm{s,out}}}{P_{\mathrm{n,out}}} \\ &= \frac{|\sum w_m^* r_m|^2}{P_{\mathrm{n}} \sum |w_m|^2} \\ &= \frac{1}{P_{\mathrm{n}}} \frac{\mathbf{w}^H \mathbf{r}\, \mathbf{r}^H \mathbf{w}}{\mathbf{w}^H \mathbf{w}} \end{aligned} \tag{11.17}$$

From (7.38), the SNR is maximized if

$$\mathbf{w} = \mathbf{r} \tag{11.18}$$

which means that for maximum SNR we weight each branch by the signal amplitude. The output SNR for these weights is

$$\begin{aligned}\gamma_{\text{out,max}} &= \frac{\mathbf{r}^H\mathbf{r}}{P_\text{n}} \\ &= \sum_{m=1}^{M} \gamma_m\end{aligned}$$

For a Rayleigh channel, the output SNR is the sum of M squared Gaussian random variables, which has a Chi-squared PDF:

$$f(\gamma) = \frac{\gamma^{M-1}e^{-\gamma/\Gamma}}{\Gamma^M(M-1)!}, \quad (\gamma > 0) \tag{11.19}$$

The CDF is

$$F(\gamma_0) = 1 - e^{-\gamma_0/\Gamma}\sum_{m=1}^{M}\frac{(\gamma_0/\Gamma)^{m-1}}{(m-1)|} \tag{11.20}$$

which leads to much better performance than simple selection diversity.

11.5.3 Equal Gain Combining

A compromise between selection diversity and maximum ratio combining is to set all gains to unity, so that the output is

$$r_\text{out} = \sum_{1}^{M} r_m \tag{11.21}$$

The average output SNR is

$$\begin{aligned}\Gamma_M &= \frac{1}{2MP_\text{n}}\text{E}[|r_\text{out}|^2] \\ &= \frac{1}{2MP_\text{n}}\sum_{m,n}\text{E}[r_m r_n^*]\end{aligned}$$

If we assume that the branches are uncorrelated, then

$$\text{E}[r_m r_n^*] = \text{E}[r_m]\text{E}[r_n^*] = \left(\sqrt{\pi P_s/2}\right)^2 = \frac{\pi}{2}P_s, \quad (m \neq n) \tag{11.22}$$

where we have assumed that the signals have been cophased and used the fact that the expected value of a Rayleigh distributed random variable with parameter σ is $\sqrt{\pi/2}\sigma$. The signal power is $P_s = \text{E}[|r_n|^2]/2 = \sigma^2$. Using this result, the average SNR is found to be

$$\begin{aligned}\Gamma_M &= \frac{1}{2MP_\text{n}}\left[2MP_s + (M^2 - M)\frac{\pi P_s}{2}\right] \\ &= \frac{P_s}{P_\text{n}}\left[1 + \frac{(M-1)\pi}{4}\right] \\ &= \Gamma\left[1 + \frac{(M-1)\pi}{4}\right]\end{aligned} \tag{11.23}$$

where the second term in the square brackets represents the relative SNR improvement due to diversity. Equal gain combining provides performance that is close to that of the optimal maximum ratio combining, but is simpler to implement and does not require adaptive weighting of the branches.